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On mass dependences of the one-loop effective action in simple backgrounds

O-Kab Kwon[1] and Choonkyu Lee[2]

Physics Department and Center for Theoretical Physics, Seoul National University

Seoul 151-742, Korea

Hyunsoo Min[3]

Physics Department, University of Seoul, Seoul 130-743, Korea

Abstract

If background fields are soft on the scale set by mass of the particle involved, a reliable approximation to the field-theoretic one-loop effective action is obtained by a systematic large mass expansion involving higher-order Seeley-DeWitt coefficients. Moreover, if the small mass limit of the effective action in a particular background has been found by some other means, the two informations may be used to infer the corresponding result for *arbitrary* mass values. This method is used to estimate the one-loop contribution to the QCD vacuum tunneling amplitude by quarks of arbitrary mass.

[1]Electronic address; kok@phya.snu.ac.kr
[2]Electronic address; cklee@phya.snu.ac.kr
[3]Electronic address; hsmin@dirac.uos.ac.kr

Quantum or loop corrections to the effective action, in general or specific background fields, are of fundamental importance in field-theoretic studies of many physical processes. The most well-known example is the exact one-loop QED effective action for electrons in a uniform electromagnetic field background, computed first by Euler and Heisenberg[1] and many others since [2, 3]: this provides us with valuable information on the vacuum polarization phenomenon and on the electron-positron pair production from the vacuum. Also interesting physical effects have been demonstrated by studying the one-loop correction to the effective action in a soliton or instanton background [4, 5].

In four-dimensional field theory contexts, however, the *exact* computation of the one-loop effective action in any non-trivial background field generally corresponds to a formidable mathematical problem. A well-known approximation scheme in this regard is the so-called derivative expansion[3, 6] of the effective action, which may be used for a sufficiently smooth background field. In this paper, we discuss the possibility of utilizing a large mass expansion (for which simple computer algorithms have been developed recently) and mass interpolation to find the one-loop effective action for an arbitrary mass parameter. Euclidean four-dimensional space-time is assumed below.

To explain our approach, consider the one-loop effective action $\Gamma(A)$ for a complex spin-0 field of mass m in some Yang-Mills background fields $A_\mu^a(x)$. The quadratic differential operator appropriate to the scalar field is

$$G^{-1} + m^2 = -D_\mu D_\mu + m^2 \quad (\equiv -D^2 + m^2) \tag{1}$$

(with $D_\mu = \partial_\mu - iA_\mu^a T^a \equiv \partial_\mu - iA_\mu$), and the corresponding background-free one is $G_0^{-1} + m^2 = -\partial^2 + m^2$. The Pauli-Villars regularized form of the effective action can then be expressed as

$$\Gamma(A) \;=\; \ln\left[\frac{\mathrm{Det}(G^{-1}+m^2)}{\mathrm{Det}(G_0^{-1}+m^2)}\frac{\mathrm{Det}(G_0^{-1}+\Lambda^2)}{\mathrm{Det}(G^{-1}+\Lambda^2)}\right]$$

$$= \; -\int_0^\infty \frac{ds}{s}(e^{-m^2 s}-e^{-\Lambda^2 s})\int d^4x \,\mathrm{tr}\left[<xs|x> - <xs|x>|_{A_\mu=0}\right]. \qquad (2)$$

Here the second expression is the Schwinger proper-time representation[2] which involves the coincidence limit of the proper-time Green function, $<xs|y>\equiv<x|e^{-sG^{-1}}|y>$. The latter admits the small-s asymptotic expansion of the form [7, 8]

$$s\to 0+: \qquad <xs|y>=\frac{1}{(4\pi s)^2}e^{-\frac{(x-y)^2}{4s}}\left\{\sum_{n=0}^{\infty}s^n a_n(x,y)\right\}, \qquad (3)$$

with $a_0(x,x)=1$.

Using the expansion (3) in (2), one finds that the divergent terms of $\Gamma(A)$ as $\Lambda\to\infty$ are related to the first and second Seeley-DeWitt coefficients, $\tilde{a}_1(x)\equiv \mathrm{tr}a_1(x,x)$ and $\tilde{a}_2(x)\equiv \mathrm{tr}a_2(x,x)$. Simple calculations yield

$$\tilde{a}_1(x)=0, \qquad \tilde{a}_2(x)=-\frac{1}{12}\mathrm{tr}(F_{\mu\nu}(x)F_{\mu\nu}(x)), \qquad (4)$$

where $F_{\mu\nu}\equiv F^a_{\mu\nu}T^a=i[D_\mu,D_\nu]$. Hence the above effective action can be cast as

$$\Gamma(A)=\frac{1}{12}\frac{C}{(4\pi)^2}(\ln\frac{\Lambda^2}{m^2})\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}+\overline{\Gamma}(A), \qquad (5)$$

(C is defined by $\mathrm{tr}(T^aT^b)=\delta_{ab}C$), where the amplitude

$$\overline{\Gamma}(A)=-\int_0^\infty\frac{ds}{s^3}e^{-m^2s}\int d^4x\left[1-\left(1+s\frac{\partial}{\partial s}+\frac{1}{2}s^2\frac{\partial^2}{\partial s^2}\right)\Big|_{s=0}\right]\mathrm{tr}\left(s^2<xs|x>\right) \qquad (6)$$

is well-defined as long as $m^2\neq 0$. [In (6), $(1+s\frac{\partial}{\partial s}+\frac{1}{2}s^2\frac{\partial^2}{\partial s^2})|_{s=0}f(s)\equiv f(0)+sf'(0)+\frac{1}{2}s^2f''(0)$]. The logarithmic divergence in (5) is canceled by the renormalization counterterm associated with the coupling constant renormalization of the classical action $\frac{1}{4g_0^2}\int d^4xF^a_{\mu\nu}F^a_{\mu\nu}$. But the result is renormalization-prescription dependent. In fact, our amplitude $\overline{\Gamma}(A)$ in (6) can be viewed as *a* renormalized one-loop effective action for $m^2\neq 0$. If one instead adds to $\Gamma(A)$ the counterterm $\Delta\Gamma(A)=-\frac{1}{12}\frac{C}{(4\pi)^2}(\ln\frac{\Lambda^2}{\mu^2})\int d^4xF^a_{\mu\nu}F^a_{\mu\nu}$ (μ is an arbitrarily introduced renormalization mass), the resulting renormalized one-loop effective action reads

$$\Gamma_{\mathrm{ren}}(A)=-\frac{1}{12}\frac{C}{(4\pi)^2}(\ln\frac{m^2}{\mu^2})\int d^4xF^a_{\mu\nu}F^a_{\mu\nu}+\overline{\Gamma}(A), \qquad (7)$$

which has now a well-defined limit even for $m^2 \to 0$. For the expression in the minimal subtraction[9] in the dimensional regularization scheme, a further finite renormalization counterterm should be introduced[5]. These differences in the renormalized expressions reflect different ways of defining the renormalized coupling.

The next task will be to find the full finite amplitude for the one-loop effective action; for any non-trivial background field, this is very difficult. If the mass m is relatively large, however, a large-mass expansion obtained by inserting the asymptotic series (3) into (6) can be useful:

$$\overline{\Gamma}(A) = -\frac{1}{(4\pi)^2} \sum_{n=3}^{\infty} \frac{(n-3)!}{(m^2)^{n-2}} \int d^4x \tilde{a}_n(x), \qquad (\tilde{a}_n(x) \equiv \mathrm{tr} a_n(x,x)). \tag{8}$$

For $\Gamma_{\mathrm{ren}}(A)$, one may use the formula (7) together with this expansion. Thus, for relatively large mass, the one-loop effective action can be approximated by a series involving higher-order Seeley-DeWitt coefficients $\tilde{a}_n(x)$ ($n \geq 3$), for which computer algorithms are now available[10]. The useful range of this expansion, as regards the magnitude of m, will depend much on the nature of the background field and on the characteristic scale entering the background.

In this work we are interested in the one-loop effective action in some physically important background as a *function of mass parameter* m. Even for a simple background field, it will not be possible to infer the complete m-dependence on the basis of the series(8) alone; the series loses the predictive power for 'small' values of m. Actually, as we shall see below, this large mass expansion (truncated at certain order) appears to provide a surprisingly good approximation even for only moderately large values of m. Then, if the effective action in the small mass limit became known by independent methods (possibly exploiting certain symmetry present in the zero mass case), one may hope that a reliable interpolation between the small-mass and relatively large-mass expressions could be made to obtain a reasonable fit over the entire mass values. Below, we shall first test this idea

with the constant Yang-Mills field strength background case for which the exact one-loop effective action is known. The same method will then be applied to the case of significant interest—we estimate the one-loop instanton contribution to the QCD vacuum tunneling [5, 11] by quarks of arbitrary mass. The QCD vacuum tunneling amplitude due to quarks of vanishingly small mass was calculated analytically by 'tHooft[5]; this result may be relevant to u- and d-quarks, but not for others.

In the case of non-Abelian gauge theories, a constant field strength can be realized either by an Abelian vector potential which varies linearly with x^μ or by a constant vector potential whose components do not commute[12]. In this paper we only consider the case of the Abelian vector potential. Assuming SU(2) gauge group, an Abelian vector potential can then be written as $A_\mu = -\frac{1}{4} f_{\mu\nu} x_\nu \tau_3$ (with the field strength tensor $F_{\mu\nu} = f_{\mu\nu}\tau_3/2$), where τ^3 is the third Pauli matrix. If we further restrict our attention to the self-dual case, we can set $f_{23} = f_{41} = H$ with the constant 'magnetic' field H.

In this Abelian constant self-dual field, let us consider the one-loop effective action induced by isospin-1/2, spin-0 matter fields, taking the mass m of our spin-0 fields to be relatively large so that the large mass expansion (8) may be used. From the result of [10], some leading Seeley-DeWitt coefficients are easily evaluated for this case:

$$\tilde{a}_4(x) = \frac{2}{15}(H/2)^4, \quad \tilde{a}_6(x) = -\frac{4}{189}(H/2)^6, \quad \tilde{a}_8(x) = \frac{2}{675}(H/2)^8 \tag{9}$$

[Note that we get zero for all odd coefficients here]. Using these values, we then find for relatively large m the expression

$$\overline{\Gamma}(H;m) = -\frac{VH^2}{8\pi^2}\left(\frac{1}{240}(\frac{H}{m^2})^2 - \frac{1}{1008}(\frac{H}{m^2})^4 + \frac{1}{1440}(\frac{H}{m^2})^6 + \cdots\right), \tag{10}$$

where V denotes the four-dimensional Euclidean volume. For this case, it is actually not difficult to find the *exact* expression for the one-loop effective action, following rather

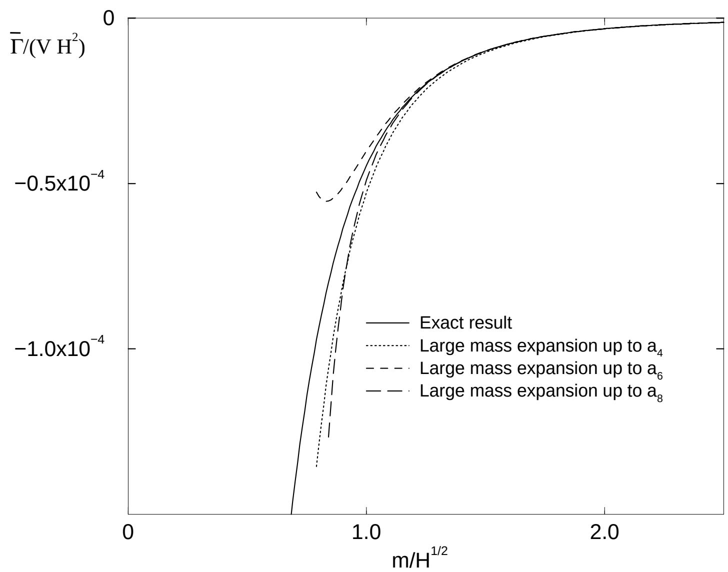

Figure 1: Plot of the effective action $\overline{\Gamma}(H;m)$.

closely Schwinger's original analysis in QED[2]: the result for $\overline{\Gamma}$ (see (6)) turns out to be

$$\overline{\Gamma}(H;m) = -2V \int_0^\infty \frac{ds}{s} e^{-m^2 s} \frac{1}{(4\pi s)^2} \left[\frac{(Hs/2)^2}{\sinh^2(Hs/2)} - 1 + \frac{1}{3}(Hs/2)^2 \right]. \tag{11}$$

Comparing the result of large mass expansion in (10) against this exact expression, we can investigate the validity range of the former. From the plots in Fig.1, it should be evident that for mass value in the range $m/\sqrt{H} \gtrsim 1$, summing only a few leading terms in the series (10) already produces values which are very close to the exact ones.

Now suppose that the exact expression (11) were not available to us. For mass value which is not so large (i.e., if $m/\sqrt{H} < 1$), large mass expansion (10) fails to give useful information. Nevertheless, if one happens to know the one-loop effective action for *small* mass, this additional information and the large mass expansion may well be used to infer the behavior of the effective action for general, small or large, mass. In exhibiting this, $\overline{\Gamma}(H;m)$ will not be convenient since it becomes ill-defined as $m \to 0$. So, based on the relation (7), we may consider the renormalized action $\Gamma_{\mathrm{ren}}(H;m,\mu)$ given by

$$\Gamma_{\mathrm{ren}}(H;m,\mu) = -\frac{VH^2}{(4\pi)^2 \cdot 6} \ln(\frac{m^2}{\mu^2}) + \overline{\Gamma}(H;m). \tag{12}$$

which is well-behaved for small m. Large mass expansion for $\Gamma_{\rm ren}(H;m,\mu)$ results once if the expansion (10) is substituted in the right hand side of (12). On the other hand, $\Gamma_{\rm ren}(H;m,\mu)$ has the small-m expansion (which is extracted using (11)),

$$\Gamma_{\rm ren}(H;m=0,\mu) = VH^2\left(\frac{1}{(4\pi)\cdot 6}\ln\frac{\mu^2}{H} + 0.00209 - 0.00633(\frac{m^2}{H}) + \dots\right). \qquad (13)$$

In combining these two informations from different mass ranges, it is convenient to consider the μ-independent quantity (especially for mass interpolation purpose)

$$\tilde{\Gamma}(H;m) \equiv \Gamma_{\rm ren}(H;m,\mu) - \Gamma_{\rm ren}(H;m=0,\mu). \qquad (14)$$

In Fig.2, the graph for $\tilde{\Gamma}(H;m)$ has been given as a function of $m/\sqrt{H}$. The exact result, represented by a solid line, exhibits a monotonically decreasing behavior starting from the maximum at $m/\sqrt{H}=0$. As we mentioned already, the large mass expansion can be trusted in the range $m/\sqrt{H} \gtrsim 1$. This curve may then be smoothly connected to that given from the small-m expansion (13), assuming a monotonic behavior (as should be reasonable for a simple background field). Evidently, with this interpolation, one could have acquired a nice overall fit over the entire mass range if the exact curve were not known.

Now turn to the case of a BPST instanton background[13], i.e., a self-dual solution of Yang-Mills field equations given by

$$A_\mu(x) \equiv A_\mu^a(x)\frac{\tau^a}{2} = \frac{\overset{(+)}{\eta}_{\mu\nu a}\tau_a x_\nu}{x^2+\rho^2}, \qquad (15)$$

where $\overset{(+)}{\eta}_{\mu\nu a}$ $(a=1,2,3)$ are the so-called 'tHooft symbols[5]. With QCD in mind, the effective action due to a spin-1/2 quark field (in the fundamental representation) with unspecified mass m will be of special interest. Here we define the proper-time Green function by $< xs|y > = < x|e^{-s(\gamma_\mu D_\mu)^2}|y >$ (our antihermitian γ-matrices satisfy the relations $\{\gamma_\mu,\gamma_\nu\} = -2\delta_{\mu\nu}$), so that we may have the spin-1/2 one-loop effective action

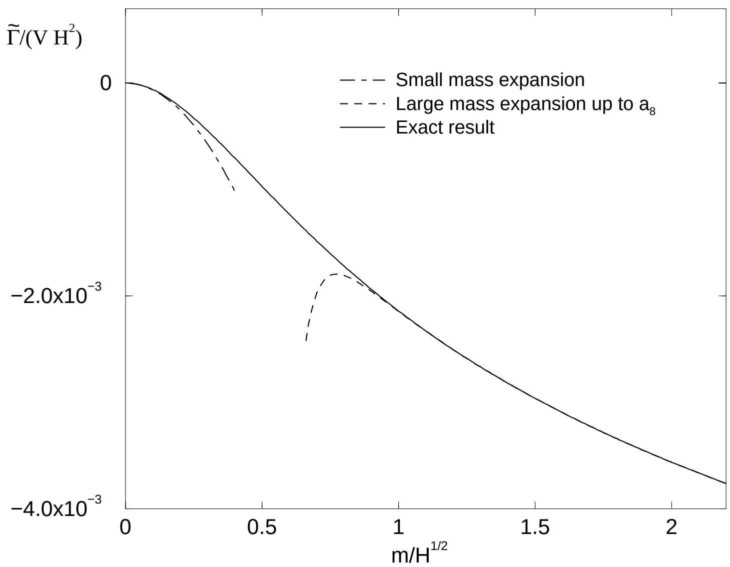

Figure 2: Plot of $\tilde{\Gamma}(H;m)$.

expressed as

$$\Gamma^{(1/2)}(A) = \frac{1}{2}\int_0^\infty \frac{ds}{s}(e^{-m^2 s} - e^{-\Lambda^2 s}) \int d^4x \, \mathrm{tr}\left[< xs|x > - < xs|x > |_{A_\mu=0}\right]. \qquad (16)$$

For the corresponding $\tilde{a}_2$-coefficient, we have $\tilde{a}_2 = \frac{2}{3}\mathrm{tr}(F_{\mu\nu}(x)F_{\mu\nu}(x))$. So the renormalized one-loop effective action $\Gamma^{(1/2)}_{\rm ren}(A)$—the direct spin-1/2 analogue of (7)—can be obtained if the counterterm $\Delta\Gamma(A) = -\frac{1}{3}\frac{C}{(4\pi)^2}(\ln\frac{\Lambda^2}{\mu^2})\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}$ is added to the unrenormalized expression (16). Note that if the Dirac operator $\gamma_\mu D_\mu$ possesses normalizable zero modes [14], the renormalized quantity $\Gamma^{(1/2)}_{\rm ren}(A)$ is still infrared divergent at $m^2 = 0$. Actually, based on the hidden supresymmetry present in a self-dual Yang-Mills background[5], it is possible to derive a following simple relationship[15] existing between the spin-1/2 and spin-0 one-loop effective actions:

$$\Gamma^{(1/2)}_{\rm ren}(A) = -\frac{1}{2}n_F\left(\ln\frac{m^2}{\mu^2}\right) - 2\Gamma_{\rm ren}(A), \qquad (17)$$

or, for the respective contributions to the tunneling amplitude,

$$e^{-\Gamma^{(1/2)}_{\rm ren}(A)} = \left(\frac{m}{\mu}\right)^{n_F} e^{2\Gamma_{\rm ren}(A)}. \qquad (18)$$

Here, n_F is the number of normalizable spinor zero modes in the given self-dual Yang-Mills background, and $\Gamma_{\rm ren}(A)$ the corresponding one-loop effective action(defined in accordance with (7) above) for a 'spin-0 quark' of the same mass m. Due to this relationship, our problem is again reduced to that of a spin-0 field.(To obtain the spin-1/2 one-loop effective action in the minimal subtraction in the dimensional regularization scheme, the finite renormalization counterterm $\Delta\Gamma(A)' = \frac{C}{(4\pi)^2\cdot 3}(\ln 4\pi - \gamma)\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}$ ($\gamma = 0.5772\ldots$ is the Euler's constant) must be added further to that of $\Gamma^{(1/2)}_{\rm ren}(A)$[5].)

For relatively large mass m, the renormalized effective action for a spin-0 matter field in the instanton background (15) can be studied with the help of large-mass asymptotic series (8) for $\overline{\Gamma}$. Note that $\overline{\Gamma}$ is a function of $m\rho$ only. The coefficient $a_3(x,x)$ is

$$a_3(x,x) = \frac{1}{120}D_\mu F_{\nu\lambda}D_\mu F_{\nu\lambda} - \frac{i}{45}F_{\mu\nu}F_{\nu\lambda}F_{\lambda\mu}, \tag{19}$$

and so, for the instanton background (15), we obtain the result $\int d^4x\ \tilde{a}_3(x) = \frac{16}{75}\frac{\pi^2}{\rho^2}$. Calculations of higher-order Seeley-DeWitt coefficients with the instanton background can be very laborious.(For $a_6(x,x)$ for instance, the full expression occupies more than a page[10]). Together with the formulas given in ref.[10], we have thus used the "Mathematica" program to do the necessary trace calculations and also tensor algebra. The results are as follows:

$$\int d^4x\,\tilde{a}_4(x) = \frac{272}{735}\frac{\pi^2}{\rho^4}, \qquad \int d^4x\,\tilde{a}_5(x) = -\frac{1856}{2835}\frac{\pi^2}{\rho^6}, \qquad \int d^4x\,\tilde{a}_6(x) = \frac{63328}{444675}\frac{\pi^2}{\rho^8}. \tag{20}$$

From these we obtain the following expression for $\overline{\Gamma}(m\rho)$:

$$\overline{\Gamma}(m\rho) = -\frac{1}{75}\frac{1}{m^2\rho^2} - \frac{17}{735}\frac{1}{m^4\rho^4} + \frac{232}{2835}\frac{1}{m^6\rho^6} - \frac{7916}{148225}\frac{1}{m^8\rho^8} + \cdots \tag{21}$$

Plotting this expression (first keeping only the a_3-term, then including the a_4-term also, etc), we find that the curve is quite stable if $m\rho \gtrsim 1.5$. (See Fig.3). The result of large mass expansion may thus be trusted in the mass range given by $m\rho \gtrsim 1.5$.

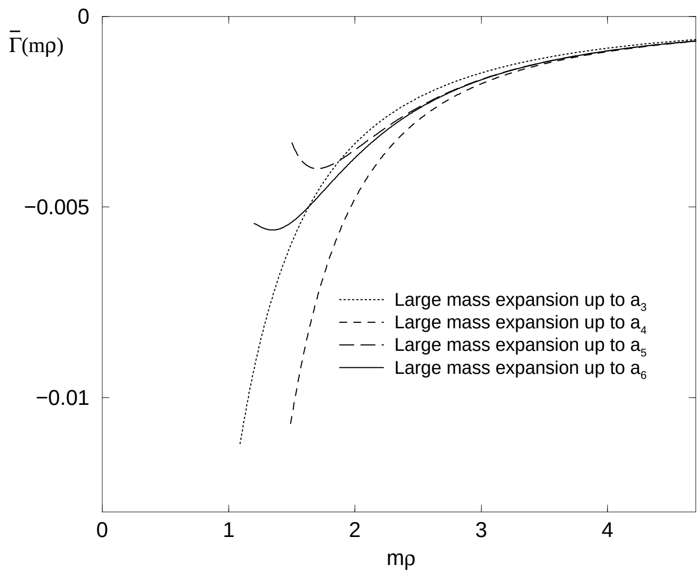

Figure 3: Plot of $\overline{\Gamma}(m\rho)$ for the instanton background.

To compare the above findings with the small-mass expression, we again consider the renormalized effective action $\Gamma_{\rm ren}(A)$, which is denoted in the instanton background (15) by $\Gamma_{\rm ren}(m,\rho,\mu)$. From (7), we have

$$\Gamma_{\rm ren}(m,\rho,\mu) = -\frac{1}{6}\ln\frac{m}{\mu} + \overline{\Gamma}(m\rho). \tag{22}$$

Then, from the computations of 'tHooft[5] and of ref.[16], $\Gamma_{\rm ren}(m,\rho,\mu)$ for sufficiently small values of $m\rho$ is approximated by

$$\Gamma_{\rm ren}(m,\rho,\mu) = \frac{1}{6}\ln\mu\rho + \alpha(\frac{1}{2}) + \frac{1}{2}(m\rho)^2\ln m\rho + \cdots \tag{23}$$

with $\alpha(\frac{1}{2}) = \frac{1}{6}\gamma + \frac{1}{6}\ln\pi - \frac{1}{\pi^2}\zeta'(2) - \frac{17}{72}$, where $\zeta'(s)$ is the first derivative of Riemann zeta function. We also define the μ-independent quantity

$$\begin{aligned}\tilde{\Gamma}(m\rho) &\equiv \Gamma_{\rm ren}(m,\rho,\mu) - \Gamma_{\rm ren}(0,\rho,\mu) \\ &= -\frac{1}{6}\ln m\rho - \alpha(\frac{1}{2}) + \overline{\Gamma}(m\rho). \end{aligned}\tag{24}$$

For sufficiently small $m\rho$, we have $\tilde{\Gamma}(m\rho) \simeq \frac{1}{2}(m\rho)^2\ln m\rho$; but, for $m\rho \gtrsim 1.5$, a good approximation to $\tilde{\Gamma}(m\rho)$ results if (21) is used in the second form of (24). These small-mass

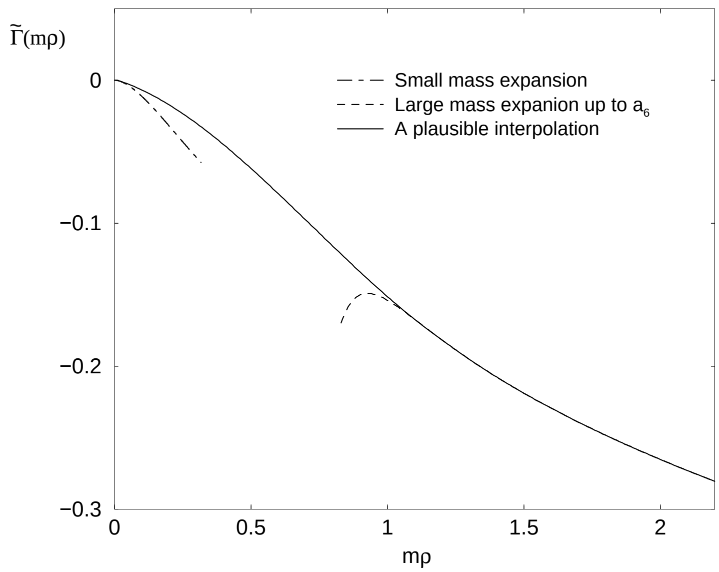

Figure 4: Plot of $\tilde{\Gamma}(m\rho)$ for the instanton background.

and relatively large-mass expressions for $\tilde{\Gamma}(m\rho)$ are plotted in Fig.4. Also included is the smooth interpolating curve connecting the two regions, assuming the monotonousness in the range $0 < m\rho \lesssim 1.5$. In view of a simple character of the background field (15) and the fact that $m\rho$ is the sole relevant variable for $\tilde{\Gamma}$, we believe that the latter assumption is very plausible. For further support on this, we need an improved approximation(i.e., beyond (23)) for small mass; this is left for future investigation. Besed on this interpolation, one might also go on to devise a simple functional form for $\Gamma_{\rm ren}(m, \rho, \mu)$(approximately valid for *any* mass value) for phenomenological studies concerning instanton effects.

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